Apportionment Problems
Lecture 20
Section 4.1

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Outline

Apportioning Candies

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- So, how many candies should each student get?

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- Why not?

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- For example, if State A has twice the population of State B, then State A should have twice as many seats as State B.

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- How many seats should each state get?
- $323{,}000{,}000 \div 435 = 742{,}529.$
- Each seat should represent 742,529 people.

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- So, Virginia gets 11.329 seats.
- Obviously, that is not possible, either.
- What to do?

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  - Separate each of those numbers into a whole number and the fractional part.
  - Give each state its whole number of seats.
  - Distribute the remaining seats to those states with the largest fractional parts.

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- Congress failed to override Washington's veto.
- What happened next?

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Assignment

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- Chapter 4: Exercises 11, 12, 13, 14. Skip the terminology; apply Hamilton's method.